Contents

Preface **xi**

1 Algorithm Analysis **1**
1.1 Analyzing Algorithms . . . 3
1.2 A Quick Mathematical Review . . . 19
1.3 A Case Study in Algorithm Analysis . . . 29
1.4 Amortization . . . 34
1.5 Exercises . . . 42

Part I: Data Structures

2 Basic Data Structures **51**
2.1 Stacks and Queues . . . 53
2.2 Lists . . . 60
2.3 Trees . . . 68
2.4 Exercises . . . 84

3 Binary Search Trees **89**
3.1 Searches and Updates . . . 91
3.2 Range Queries . . . 101
3.3 Index-Based Searching . . . 104
3.4 Randomly Constructed Search Trees . . . 107
3.5 Exercises . . . 110

4 Balanced Binary Search Trees **115**
4.1 Ranks and Rotations . . . 117
4.2 AVL Trees . . . 120
4.3 Red-Black Trees . . . 126
4.4 Weak AVL Trees . . . 130
4.5 Splay Trees . . . 139
4.6 Exercises . . . 149

5 Priority Queues and Heaps **155**
5.1 Priority Queues . . . 157
5.2 PQ-Sort, Selection-Sort, and Insertion-Sort . . . 158
5.3 Heaps . . . 163
5.4 Heap-Sort . . . 174
5.5 Extending Priority Queues . . . 179
5.6 Exercises . . . 182

6 Hash Tables **187**
6.1 Maps . . . 189
6.2 Hash Functions . . . 192
6.3 Handling Collisions and Rehashing . . . 198
6.4 Cuckoo Hashing . . . 206
6.5 Universal Hashing . . . 212
6.6 Exercises . . . 215

7 Union-Find Structures **219**
7.1 Union-Find and Its Applications . . . 221
7.2 A List-Based Implementation . . . 225
7.3 A Tree-Based Implementation . . . 228
7.4 Exercises . . . 236

Part II: Sorting and Selection

8 Merge-Sort and Quick-Sort **241**
8.1 Merge-Sort . . . 243
8.2 Quick-Sort . . . 250
8.3 A Lower Bound on Comparison-Based Sorting . . . 257
8.4 Exercises . . . 259

9 Fast Sorting and Selection **265**
9.1 Bucket-Sort and Radix-Sort . . . 267
9.2 Selection . . . 270
9.3 Weighted Medians . . . 276
9.4 Exercises . . . 279

Part III: Fundamental Techniques

10 The Greedy Method **283**
10.1 The Fractional Knapsack Problem . . . 286
10.2 Task Scheduling . . . 289
10.3 Text Compression and Huffman Coding . . . 292
10.4 Exercises . . . 298

11 Divide-and-Conquer **303**
11.1 Recurrences and the Master Theorem . . . 305
11.2 Integer Multiplication . . . 313
11.3 Matrix Multiplication . . . 315
11.4 The Maxima-Set Problem . . . 317
11.5 Exercises . . . 319

12 Dynamic Programming 323
12.1 Matrix Chain-Products 325
12.2 The General Technique 329
12.3 Telescope Scheduling 331
12.4 Game Strategies 334
12.5 The Longest Common Subsequence Problem 339
12.6 The 0-1 Knapsack Problem 343
12.7 Exercises 346

13 Graphs and Traversals 353
13.1 Graph Terminology and Representations 355
13.2 Depth-First Search 365
13.3 Breadth-First Search 370
13.4 Directed Graphs 373
13.5 Biconnected Components 386
13.6 Exercises 392

Part IV: Graph Algorithms

14 Shortest Paths 397
14.1 Single-Source Shortest Paths 399
14.2 Dijkstra's Algorithm 400
14.3 The Bellman-Ford Algorithm 407
14.4 Shortest Paths in Directed Acyclic Graphs 410
14.5 All-Pairs Shortest Paths 412
14.6 Exercises 418

15 Minimum Spanning Trees 423
15.1 Properties of Minimum Spanning Trees 425
15.2 Kruskal's Algorithm 428
15.3 The Prim-Jarník Algorithm 433
15.4 Barůvka's Algorithm 436
15.5 Exercises 439

16 Network Flow and Matching 443
16.1 Flows and Cuts 445
16.2 Maximum Flow Algorithms 452
16.3 Maximum Bipartite Matching 458
16.4 Baseball Elimination 460
16.5 Minimum-Cost Flow 462
16.6 Exercises 469

Part V: Computational Intractability

17 *NP*-Completeness **473**
17.1 ***P*** and ***NP*** 476
17.2 ***NP***-Completeness 483
17.3 CNF-SAT and 3SAT 489
17.4 VERTEX-COVER, CLIQUE, and SET-COVER 492
17.5 SUBSET-SUM and KNAPSACK 496
17.6 HAMILTONIAN-CYCLE and TSP 499
17.7 Exercises 502

18 Approximation Algorithms **507**
18.1 The Metric Traveling Salesperson Problem 511
18.2 Approximations for Covering Problems 515
18.3 Polynomial-Time Approximation Schemes 518
18.4 Backtracking and Branch-and-Bound 521
18.5 Exercises 525

Part VI: Additional Topics

19 Randomized Algorithms **529**
19.1 Generating Random Permutations 531
19.2 Stable Marriages and Coupon Collecting 534
19.3 Minimum Cuts 539
19.4 Finding Prime Numbers 546
19.5 Chernoff Bounds 551
19.6 Skip Lists 557
19.7 Exercises 563

20 B-Trees and External Memory **569**
20.1 External Memory 571
20.2 (2,4) Trees and B-Trees 574
20.3 External-Memory Sorting 590
20.4 Online Caching Algorithms 593
20.5 Exercises 600

21 Multidimensional Searching **603**
21.1 Range Trees 605
21.2 Priority Search Trees 609
21.3 Quadtrees and k-d Trees 614
21.4 Exercises 618

22 Computational Geometry 623
22.1 Operations on Geometric Objects . . . 625
22.2 Convex Hulls . . . 630
22.3 Segment Intersection . . . 638
22.4 Finding a Closest Pair of Points . . . 642
22.5 Exercises . . . 646

23 String Algorithms 651
23.1 String Operations . . . 653
23.2 The Boyer-Moore Algorithm . . . 656
23.3 The Knuth-Morris-Pratt Algorithm . . . 660
23.4 Hash-Based Lexicon Matching . . . 664
23.5 Tries . . . 669
23.6 Exercises . . . 680

24 Cryptography 685
24.1 Greatest Common Divisors (GCD) . . . 687
24.2 Modular Arithmetic . . . 691
24.3 Cryptographic Operations . . . 699
24.4 The RSA Cryptosystem . . . 703
24.5 The El Gamal Cryptosystem . . . 706
24.6 Exercises . . . 708

25 The Fast Fourier Transform 711
25.1 Convolution . . . 713
25.2 Primitive Roots of Unity . . . 715
25.3 The Discrete Fourier Transform . . . 717
25.4 The Fast Fourier Transform Algorithm . . . 721
25.5 Exercises . . . 727

26 Linear Programming 731
26.1 Formulating the Problem . . . 734
26.2 The Simplex Method . . . 739
26.3 Duality . . . 746
26.4 Applications of Linear Programming . . . 750
26.5 Exercises . . . 753

A Useful Mathematical Facts 761
Bibliography 765
Index 774